Linear Algebra
Introduction

0011 0010 1010 1101 0001 0100 1011

Dr.A.L.Pathak
Department of Mathematics
Brahmanand PG College,The Mall,Kanpur

What is Linear Algebra?

0011 0010 1010 1101 0001 0100 1011

- develops from the idea of trying to solve and analyze systems of linear equations.
- theory of matrices and determinants arise from this effort
- we will then generalize these ideas to the abstract concept of a vector space
- here we will look at linear transformations, eigenvalues, inner products...

Why is Linear Algebra interesting?

0011 0010 1010 1101 0001 0100 1011

- It has many applications in many diverse fields. (computer graphics, chemistry, biology, differential equations, economics, business, ...)
- It strikes a nice balance between computation and theory.
- Great area in which to use technology (Maple).

What is a linear equation?

- A **linear equation** is an equation of the form, $a_n x_n + a_{n-1} x_{n-1} + \ldots + a_1 x_1 = b$.
- A **solution** to a linear equation is an assignment of values to the variables (x_i's) that make the equation true.

What is a system of linear equations?

- A **system of linear equations** is simply a set of linear equations. i.e.

$$a_{1,1}x_1 + a_{1,2}x_2 + \ldots + a_{1,n}x_n = b_1$$

$$a_{2,1}x_1 + a_{2,2}x_2 + \ldots + a_{2,n}x_n = b_2$$

$$\ldots$$

$$a_{m,1}x_1 + a_{m,2}x_2 + \ldots + a_{m,n}x_n = b_m$$

- A **solution** to a system of equations is simply an assignment of values to the variables that satisfies (is a solution to) all of the equations in the system.
- If a system of equations has at least one solution, we say it is **consistent**.
- If a system does not have any solutions we say that it is **inconsistent**.

Examples

$$x + 3y = 4$$
$$2x - y = 1$$

This is a consistent system as $x = 1$, $y = 1$ is a solution.

$$x + y = 2$$
$$x + y = 4$$

This is an inconsistent system. Why??

How many solutions does this system have?

$$x + 3y = 4$$

$$-2x - 6y = -8$$

How many solutions could a system have?

- Let's look at this graphically in Maple.
- As we saw, a system of linear equations can have no solutions, one solution, or infinitely many solutions.
- We also saw that graphing things is one way to find the solutions.
- However, this is difficult in more than three variables.

Solving systems algebraically

0011 0010 1010 1101 0001 0100 1011

- Consider the following system,

$$\begin{aligned} 2x + 3y - z &= 5 \\ y - z &= -1 \\ z &= 3 \end{aligned}$$

What are its solution(s)?

- We say that a system in this form is in **row - echelon form.**
- We can find the solutions to a system in row-echelon form using **back substitution.**

Solving a system not in r-e form.

0011 0010 1010 1101 0001 0100 1011

- We say that two systems of equations (or equations) are **equivalent**, if they have exactly the same solutions.
- One method of solving a system of equations is to transform it to an equivalent system in r-e form.

Operations that lead to equivalent systems

1. Interchange two equations.

2. Multiply an equation by a nonzero constant.

3. Add a multiple of an equation to another equation.

- Using these three operations we can solve a system of linear equations in the following manner.

1. Save the *x* term in the first equation and use it to eliminate all other *x* terms.
2. Ignore the first equation and use the second term in the second equation to eliminate all other second terms in the remaining equations.
3. Continue in this manner until the system is in r-e form.

Writing infinite solutions in parametric form.

- Homework: p.11: 7,13,15,19,21,23, 33, 35, 45,46, 51, 61, 65